Ecole Doctorale des Sciences Fondamentales

Title of the thesis: Projective geometry and fundamental group for three-dimensional manifolds

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Summary : A hyperbolic structure on a three-dimensional manifold is determined by a discrete and faithful representation of the fundamental group of the manifold in the group of Lorentzian transformations **PO**(3,1). If the manifold is of finite volume, then this representation (called holonomy) is unique up to conjugation (Mostow rigidity). By composing the holonomy with the canonical inclusion of **PO**(3,1) in **PGL**(4, **R**), we obtain a new discrete and faithful representation $\mathbf{r_h}$ of the fundamental group in the group **PGL**(4, **R**) of the transformations of l projective space $\mathbf{R}P^3$. In this situation, Mostow's rigidity no longer applies and deformations of $\mathbf{r_h}$ give rise to new projective convex structures on the manifold.

The objective of this thesis topic is to study the deformations of projective structures on three-dimensional hyperbolic manifolds and orbifolds. Following the work of Benoist and Koszul we are led, in a first time, to study the space of representations modulo conjugation of the fundamental group of the manifold in **PGL**(4, **R**), and in particular to understand the connected component of $\mathbf{r_h}$ in this space. It is known that in some cases this connected component is reduced to a singleton (see [1]) and in other cases it is homeomorphic to an open ball (see [2]).

References:

[1] M. Heusener and J. Porti. Infinitesimal projective rigidity under Dehn filling. Geometry & Topology, 15(4):2017–2071, 2011.

[2] L. Marquis. Espace des modules de certains polyèdres projectifs miroirs. Geom. Dedicata, 147:47–86, 2010.